Exercise 5 – 30.10.2025

Modified Cam-clay (MCC) model

Problem statement

In this exercise, we will use the Modified Cam-clay (MCC) model to predict the behavior of a clay sample.

The clay sample was isotropically compressed to a $p'_0 = 225\ kPa$ and then unloaded to $p'_a = 150\ kPa$ at which the void ratio was $e_a = 1.4$. Starting from p'_a a drained CTC was then performed.

We want to predict the strains that the sample will undergo during the test when the deviatoric stress q increases by $12\ kPa$ after reaching the initial yield surface.

The clay parameters have previous been determined:

$$\lambda = 0.16, \kappa = 0.05, \varphi'_{cv} = 25.5°, \nu = 0.3$$

The Modified Cam-clay (MCC) model is used to predict the clay sample behavior.

Question 1 – Initial strains

a. Determine the initial yield stresses: the mean effective yield stress p'_{y1} and the deviatoric yield stress q_{y1}.
b. Plot the ESP, the critical state line, and the initial yield surface in the $(q - p')$ plane.
c. Calculate the elastic volumetric and deviatoric strains increments at initial yield: $\Delta\varepsilon^e_{v,i}$ and $\Delta\varepsilon^e_{d,i}$.

Help for c. → To compute $\Delta\varepsilon^e_{d,i}$, we make the approximation of a linear elastic bulk modulus

$$K = \frac{\Delta p'(1+e)}{\kappa}$$ and we use the shear modulus $G = \frac{3K(1-2\nu)}{2(1+\nu)}$.

Question 2 – Strains after initial yield

a. Determine the yield stresses after further increasing the deviatoric stress of 12 kPa (the mean effective yield stress p'_{y2} and the deviatoric yield stress q_{y2}) and the preconsolidation mean effective stress of the new yield surface p'_{02}.
b. Calculate the total volumetric (ε_v) and deviatoric (ε_d) strain generated by the deviatoric loading.
   We consider an associated flow rule ($g = F = 0$) and the shear modulus G is assumed constant during the shearing test.
c. On the same graph, plot the new yield surface in the $(q - p')$ plane

Definitions of interest

The Modified Cam-clay model yield surface is defined as:

$$F = \frac{q^2}{M^2} + p'\left(p' - p'_0\right) = 0 \qquad (1)$$

The elastic volumetric strain increment can be written as:

$$\Delta\varepsilon_v^e = \frac{\kappa}{1+e_0} \ln \frac{p'_2}{p'_1} \qquad (2)$$

$$\Delta\varepsilon_v^e = \frac{\Delta p'}{K} \qquad (3)$$

The elastic deviatoric strain increment can be written as:

$$\Delta\varepsilon_d^e = \frac{\Delta q}{3G} \qquad (4)$$

The plastic volumetric strain increment can be written as:

$$d\varepsilon_v^p = \Lambda \frac{\delta g}{\delta p'} \qquad (5)$$

$$\Delta\varepsilon_v^p = \frac{\lambda-\kappa}{1+e_0} \ln\left(\frac{p'_{02}}{p'_{01}}\right) \qquad (6)$$

The plastic deviatoric strain increment can be written as:

$$d\varepsilon_d^p = \Lambda \frac{\delta g}{\delta q} \qquad (7)$$